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de Jong, T. [de Jong, Theo] (NL-NIJM-MI); **van Straten, D.** (D-KSRL)

On the deformation theory of rational surface singularities with reduced fundamental cycle. (English summary)

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The singularities of the title can be characterised as those for which the general hyperplane section is L_n^n: a curve isomorphic to the coordinate axes in $\mathbf{C}^n$. The surface singularity X is then a one-parameter smoothing of L_n^n. From this fact the authors derive a very simple and beautiful set of equations for X, in a nonminimal embedding, which plays an important role in this paper. As the versal deformation of L_n^n is given by equations $(y_p+a_{qp})(y_q+a_{pq})-S_{pq}=0$, where S_{pq} depends only on the deformation parameters, the surface X can be described in the coordinates z_{pq} and x by quadratic equations $z_{pq}z_{qp}-S_{pq}(x)=0$ and linear equations $z_{pr}-z_{qr}-\varphi_{pq;r}(x)=0$. The S_{pq} and $\varphi_{pq;r}$ satisfy a system of equations, coming from the equations of the base space of L_n^n.

A second description uses the resolution graph: this is a tree with the property that, if extended with arrows for each branch of the curve L_n^n, the weight of each vertex is equal to minus the number of vertices and arrowheads connected to it. The order in x of the functions S_{pq} and $\varphi_{pq;r}$ has an interpretation in the graph. The authors show, given a graph, how to find equations of all rational surface singularities with this graph. Because of the emphasis on the general hyperplane section, these equations are in general not weighted homogeneous if X admits a $\mathbf{C}^*$-action.

The main results of this paper concern the modules T^1 of infinitesimal deformations, and T^2, containing the obstructions. Explicit generators are given. The authors prove that $\dim T^1=\sum_{\nu>2}(\nu-3)+h^1(\widetilde{X},\Theta_{\widetilde{X}})$, and $\dim T^2=\sum_{\nu>2}(\nu-1)(\nu-3)$, where the sum runs over the multiplicity of all infinitely near singular points (X is absolutely isolated), and $\widetilde{X}$ is the minimal resolution. The dimension of T^1 depends on moduli, but they only influence the dimension of the equisingular stratum. The proof uses among others things a one-parameter deformation to a space having as singularities the cone over the rational normal curve of degree n and singularities on the first blow-up of X.

In the last section an algorithm for computing a versal deformation of X is given.

Reviewed by *Jan Stevens*